4th Korean-Croatian Probability Summer Camp
Dubrovnik, June 29 - July 1, 2026

Book of abstracts

Mini courses

Bruno Toaldo (University of Turin): Modeling anomalous diffusion: The Role of CTRWs and non-local dynamics

In this course, we introduce the stochastic approach to anomalous diffusion processes through continuous-time random walks (CTRWs) and non-local dynamics. We begin by presenting the fundamentals of CTRWs and analyzing their semi-Markov properties. Next, we examine scaling limits of CTRWs and discuss how these properties are preserved under the limiting procedure. Finally, we establish the connection between the limiting processes and time-nonlocal equations.

Vanja Wagner (University of Zagreb): Nonlocal operators of regional type and pure-jump processes on domains

We present an overview of several nonlocal operators of regional type, with the model example being the regional operators generalising the fractional Laplacian. The focus will be on the analysis of three classes of operators: the Dirichlet, spectral, and regional (censored). We will discuss their analytical properties, as well as associate them with appropriate pure-jump processes on domains.

Jaeyun Yi (Korea Institute For Advanced Study): Peaks of the parabolic Anderson model with white noise potential

In this lecture series, I will discuss the parabolic Anderson model (PAM), the Cauchy problem associated with the Anderson Hamiltonian operator. Here, the potential is given by white noise, a mean-zero, delta-correlated Gaussian random field. A key characteristic of the PAM is the emergence of tall peaks localized in small regions, a phenomenon inherited from the Anderson localization of the underlying operator. We will examine the asymptotic behavior of these peaks and their multifractal structure.

Due to the roughness of the noise in higher dimensions, the PAM in higher dimensions becomes a singular stochastic PDE, which requires modern solution theories. I will also introduce how to exploit heat kernel estimates and spectral analysis tailored to these singular settings. This is based on joint work with Promit Ghosal.

Talks

Magdalena Mikić (J. J. Strossmayer University of Osijek): Superpositions of CARMA processes

We define supCARMA processes as superpositions of Lévy-driven CARMA processes with respect to a Lévy basis. They can be seen as a natural extension of superpositions of Ornstein-Uhlenbeck type processes. We focus in particular on supCAR(2) processes, which can be classified into three distinct types depending on the eigenstructure of the underlying CAR(2) matrix. For each type, sufficient conditions for existence are established, together with explicit expressions for correlation functions. Different parameter distributions lead to different memory properties and process behavior, allowing for both short-range and long-range dependence, as well as oscillatory correlations. These features make supCAR(2) processes a flexible tool for modeling temporal dependence.

Posters

Ivan Biočić (University of Zagreb): *Sampling inverse subordinators and subdiffusions and applications to Dirichlet boundary problem*

In this poster, a method to exactly sample the trajectories of inverse subordinators (in the sense of the finite-dimensional distributions), jointly with the undershooting or overshooting process, is presented. The method applies to general strictly increasing subordinators. The (random) running times of these algorithms have finite moments and explicit bounds for the expectations are provided. Additionally, the Monte Carlo approximation of functionals of subdiffusive processes (in the form of time-changed Feller processes) is considered where a central limit theorem and the Berry-Esseen bounds are proved. The approximation of time-changed Itô diffusions is also studied where the strong error is explicitly evaluated. An application of our algorithms in the context of weak ergodicity breaking of subdiffusion and to Dirichlet boundary problem is also discussed. Joint work with Bruno Toaldo and Daniel E. Cedeño-Girón.

Petra Daković (University of Zagreb): *Split-sample senalized srder selection for Gaussian mixtures*

Estimating the order of a finite Gaussian mixture model is a widely studied model selection problem with applications in clustering, density estimation and unsupervised learning. Inspired by cross-validation methods, we study a split-sample penalized procedure in which the data are divided into training and validation subsets. For each candidate order, model parameters are estimated using the training sample, while model selection is based on the validation log-likelihood combined with a complexity penalty.

We establish two complementary asymptotic results. First, we show that the penalized split-sample selector asymptotically chooses an order whose best-fitting mixture model achieves the smallest Kullback–Leibler divergence from the data-generating distribution among all candidate orders. In case that true distribution belongs to the family of mixtures under study (i.e. is correctly specified), this implies the asymptotic exclusion of underfitted models. Second, in correctly specified setting, we show that overfitted models are asymptotically indistinguishable from the true model based solely on validation likelihood. An appropriate complexity penalty sequence together with additional regularity conditions resolves this asymptotic tie. Together, these results imply consistency of the proposed order estimator.

The theoretical findings are complemented by simulation studies for Gaussian mixture models under varying sample sizes and penalty choices. The simulations illustrate the finite-sample behaviour of the procedure and provide empirical support for the theoretical results. Joint work with Azra Tafro.

Domagoj Demeterfi (SOIS): *Low-rank functional tensor-train approximations of option prices*

We study functional tensor-train decompositions in a Hilbert–Schmidt framework and relate truncation error to the regularity of the target function. For Sobolev functions we recover algebraic decay in rank, while for analytic functions we establish exponential rank convergence up to an algebraic prefactor. We discuss the implications for high-dimensional option pricing functions and give numerical examples using tensor-train Chebyshev interpolation.

Milosz Krupski: *Fully nonlinear mean field games*

Mean field games are a model of dispersal of a population governed by a common law, which can be controlled by the individual members aiming to maximise their own gains. Two approaches to such problems exist: stochastic and "PDE-stic", usually kept separate. Stochastic approach offers more generality, but isn't well suited to look at nonlinear aspects of the problem or establish uniqueness of solutions. Starting with a new stochastic model, we switch to PDE tools to show both existence and uniquenes of solutions in significantly wider generality than previously studied. Joint work with Indranil Chowdhury and Espen Jakobsen.

Petra Lazić (University of Ljubljana): *On Propagation of Chaos for the Fisher-Rao Gradient Flow in Entropic Mean-Field Optimization*

We consider a class of optimization problems on the space of probability measures motivated by the mean-field approach to studying neural networks. Such problems can be solved by constructing continuous-time gradient flows that converge to the minimizer of the energy function under consideration, and then implementing discrete-time algorithms that approximate the flow. In this work, we focus on the Fisher-Rao gradient flow and we construct an interacting particle system that approximates the flow as its mean-field limit. We discuss the connection between the energy function, the gradient flow and the particle system We provide a rigorous proof of the existence and uniqueness of thus obtained kernelized flows, as well as a propagation of chaos result that provides a theoretical justification for using the corresponding kernelized particle systems as approximation algorithms in entropic mean-field optimization. By one example we demonstrate the advantages of our Fisher-Rao algorithm over the classical Wasserstein flow.

Kinga Sztonyk (Wrocław University of Science and Technology): *Intrinsic ultracontractivity of Feynman-Kac semigroups for cylindrical stable processes*

The following Schrödinger operator

$$K = K_0 + V,$$

where

$$K_0 = \sqrt{-\frac{\partial^2}{\partial x_1^2}} + \sqrt{-\frac{\partial^2}{\partial x_2^2}},$$

is an example of a nonlocal, anisotropic, singular Lévy operator. We consider potentials $V : \mathbb{R}^2 \to \mathbb{R}$ such that $V(x)$ goes to infinity as $|x| \to \infty$. The operator $-K_0$ is a generator of a process $X_t = \left(X_t^{(1)}, X_t^{(2)}\right)$, sometimes called cylindrical, such that $X_t^{(1)}$ and $X_t^{(2)}$ are independent symmetric Cauchy processes in $\mathbb{R}$.

We define the Feynman–Kac semigroup

$$T_t f(x) = \mathbb{E}^x \left(\exp\left(-\int_0^t V(X_s)\, ds \right) f(X_t) \right).$$

Operators T_t are compact for every $t > 0$. There exists an orthonormal basis $\{\phi_n\}_{n=1}^{\infty}$ in $L^2(\mathbb{R}^2)$ and a corresponding sequence of eigenvalues $\{\lambda_n\}_{n=1}^{\infty}, 0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \cdots, \lim_{n\to\infty} \lambda_n = \infty$, such that $T_t\phi_n = e^{-\lambda_n t}\phi_n$. We can assume that ϕ_1 is positive and continuous on $\mathbb{R}^2$. The main result I would like to present concerns estimates for ϕ_1 and intrinsic ultracontractivity of the semigroup T_t under certain conditions on the potential V.

Bibliography

1. Tadeusz Kulczycki, Kinga Sztonyk, *Intrinsic ultracontractivity for Schrödinger semigroups based on cylindrical fractional Laplacian on the plane*, Semigroup Forum **111** (2025), 191–226.

Ivana Valentić (University of Zagreb): *Two point excursion theory*

In (2024) Excursion theory for the Wright–Fisher diffusion paper we present an excursion theory which is an intermediate between the excursion theory from a point and the excursion theory from a set. It is an approach based on the natural properties of these processes with two boundary points from which one makes excursions. In this work we elaborate on those results from a point of view of a Moran model, a continuous Markov chain which converges to a Wright–Fisher diffusion.

Francesco Virgili (University of Turin): *Reflecting time-changed diffusions in regular unbounded domains*

In this work, we establish an existence and uniqueness result for a time-changed SDE with reflection. We provide an application of this result by studying the asymptotic mean squared displacement of a time-changed diffusion, obliquely reflected in a generalized parabolic domain.

Ivana Vojnović (University of Novi Sad): *Microlocal defect measures and wave-front sets*

Following the unpublished ideas of Folkmar Bornemann, we introduce the notion of microlocal compactness wave front sets. They describe how far a bounded sequence in a Sobolev space is from being relatively compact. We distinguish local and nonlocal variants. An important property is a connection between microlocal defect functionals and wave front sets, that is, a connection with H-measures. H-measures are introduced independently by Luc Tartar and Patrick Gérard. Microlocal defect functionals act on spaces of symbols of appropriate pseudodifferential operators, and they are associated with a given weakly convergent sequence in some Sobolev space. We will give existence theorems and main properties of these functionals. Finally, we will show that the support of the H-measure for a given weakly convergent sequence (u_n) is equal to the microlocal wave front set of that sequence.

Jacek Wszoła (Wrocław University of Science and Technology): *Generalised eigenvector expansion of infinite Toeplitz matrices*

The poster discusses the Wiener–Hopf factorisation for infinite Toeplitz matrices $T = (a_{k-l} : k, l \geq 0)$ such that the sequences $(a_k : k \geq 1)$ and $(a_{-k} : k \geq 1)$ are completely monotone. Under relatively minor further assumptions on the sequence $(a_k : k \in \mathbb{Z})$, the main result provides a generalised eigenvector expansion for T. Somewhat surprisingly, even though we do not assume that T is self-adjoint (nor a normal operator), our expression involves only *real* generalised eigenvalues, eigenvectors and co-eigenvectors.

On the probability side, Toeplitz matrices T can be interpreted as transition matrices for discrete-time random walks with jump distributions given by the sequence $(a_k : k \in \mathbb{Z})$. Our results aim to develop fluctuation theory of random walks and they will likely lead to integral formulae analogous to those obtained recently for continuous counterparts of Toeplitz matrices and random walks: the Wiener–Hopf operators and Lévy processes.

Schedule

	MONDAY 29.6.	**TUESDAY 30.6.**	**WEDNESDAY 31.6.**
8:30–9:00	Registration		
9:00–9:45	Bruno Toaldo	Vanja Wagner	Jaeyun Yi
9:50–10:35	Jaeyun Yi	Bruno Toaldo	Vanja Wagner
10:35–11:05	Coffee break	Coffee break	Coffee break
11:05–11:50	Jaeyun Yi	Bruno Toaldo	Vanja Wagner
11:55–12:40	Vanja Wagner	Jaeyun Yi	Bruno Toaldo
12:45–13:45	Lunch	Afternoon for discussion	Lunch
18:00–18:25	Magdalena Mikić		
18:30–19:30	Poster session		
19:30		Social dinner	